

The Fabrication of Knowledge in Mathematics Education:
A Postmodern Ethic toward Social Justice

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In this chapter it is my intent to draw upon a post-epistemological view of knowledge so as to reinscribe the discipline of mathematics in such a way that a more socially just manner of teaching can be recognized and embraced. Taking mathematical knowledge as constructed—fabricated—ways of knowing and thinking redefines the positionality of the teacher in relation to the learner, and the learner to others. The resulting need for the other, an ethical imperative, is established. Understanding knowledge differently, acknowledging this need for the other, and recognizing the emergence of a more just expectation for interaction—in particular educational interaction, provides the ethical dimension to this work.

As an effort to reinscribe mathematics, this chapter serves to disrupt the trend in western philosophy to reduce the Other to the Same (Atweh, Bland, & Ala'i, 2011 [this book]). Within this refusal to deny a total existence, a complete humanity (McKeith, 2011 [this book]), to the Other is the manner in which I consider the relation of social justice and education. This is a conception different than Skovsmose's (2011 [this book]) *education for social justice*, in which he emphasizes increasing a learner's life opportunities. It is also neither the *social justice in mathematics education* (equity for the subaltern) nor the *social justice through mathematics education* (fair society) identified by Atweh, Bland, and Ala'i, (2011 [this book]). It is principally a concept of social justice that emphasizes the knower's authorship and authority for knowledge, and through such authority is necessarily an actor upon the world, an actor with a need for the fully aware, vibrant, and living Other. It is a justice of knowing one's own freedom,¹ and embracing the emergent ethics of the resulting responsibility to act.

From the strong position that takes knowledge as constructed and thus embracing a new politics of truth, I shape a four-pronged orientation to teaching mathematics for social justice. Then I consider the work of teaching in order to devise a pragmatic framework through which to

¹ Stirner (1971/1845): "I myself limit myself" (p. 190). Foucault (1988/1982): We are "much freer than we feel" (p. 10).

enact a mathematical education for social justice. The chapter closes with a return to the post-epistemological view of knowledge in order to emphasize the ways that such an orientation creates a more socially just mathematics education.

Mathematical Knowledge in the Postmodern

Mathematics Education is most certainly a complex field of scientific study. It involves an academic discipline, mathematics. As a formal science, mathematics is composed of theorems derived/deduced from a set of axioms and definitions. These theorems, axioms, and definitions are taken to be the knowledge that constitutes mathematics; but not in totality. If this first way of considering mathematics is to be thought of as the content (NCTM, 2000; CCSSO & NGA, 2010) or subject matter (Harel, 2008) of mathematics, a second component that characterizes mathematical knowledge is referred to as mathematical processes (NCTM 2000; 2009), practices (CCSS, 2010), or ways of thinking (Harel, 2008). Defining mathematical knowledge as both ways of knowing *and* ways of thinking makes for a slippery notion of what in particular *knowledge* is; however, this pair wise view of thinking about the knowledge that constitutes the discipline of mathematics is most certainly a common and traditional perspective.

Questioning Dominant Views of Mathematical Knowledge

Ethnomathematics (D'Ambrosio, 1985; Powell, 2002; Powell & Frankenstein, 1997) and anti-racist (Cotton, 1990) perspectives bring to the fore that the knowledge associated with this traditional perspective is most certainly a particular knowledge having emerged from a particular culture. Furthermore, these perspectives, recognizing the sociocultural context of mathematical knowledge, emphasize the bias/hegemony associated with the emphasis on studying a select perspective, especially in the reifying of the particular knowledge of a dominant, if not oppressive, culture.

Critical Mathematics, as a way of thinking of mathematical knowledge, may be that which emerges from an idea to learn mathematics *through* critical thinking (Fawcett, 1938; NCTM, 1989). This emphasis places a priority on the democratization of ideas/knowledge

among the community of learners. Extending this broader *critical mathematics* concept to that of a Critical Mathematics Education (Alrø, Ravn, & Valero, 2010; Borba & Skovsmose, 1997; Frankenstein, 1987; Skovsmose, 1994, 2000) brings to the fore the potential of this knowledge, mathematics, to both empower or disempower students; neither result is or could be determined. In this uncertainty lies the *critical* nature of mathematics education.

Ernest (1990) argues the social constructivist perspective offers a middle path amid the traditional objective/subjective dilemma regarding the status of mathematical knowledge. “The social constructivist thesis is that mathematics is a social construction, a cultural product, fallible like any other branch of knowledge” (1990, paragraph 16). According to his social constructivism, “mathematics is more than a collection of subjective beliefs, but less than a body of absolute objective knowledge, floating above all human activity. Instead it occupies an intermediate position. Mathematics is cultural knowledge, like the rest of human knowledge. It transcends any particular individual, but not all of humankind, like art, music, literature, religion, philosophy and science” (Ernest, 2010, paragraph 6).

Each of these positions maintains (or at best toys at the edges of) an *a priori* mathematical knowledge, independent of the knowing subject. Mathematics is taken as objective, mind-independent knowledge. Rather than rehearse some absolutist, pluralist, or fallibilist vs. relativist interpretations of the possibilities for mathematical knowledge, I prefer to take on the postmodern deconstructive (Spivak, 1974) move to consider the binary suggested by the opposition of possibilities for knowledge. Stated another way, I don’t know that one could make a justified claim that any of these positions on knowledge is true. “Deconstruction does not say there is no subject, there is no truth, there is no history. It simply questions the privileging of identity so that someone is believed to have the truth. It is not the exposure of error” (p. 28). It seems that, in social interaction, we speak of a mathematics, some body of knowledge characterized by these ways of knowing and ways of thinking. And we speak of this knowledge in a manner taken to be shared. In that we desire for this knowledge to be a truth

may speak to our biological drive to have ways of knowing that remain viable within our experiential realm. The postmodern move of deconstruction, to interrogate the role of knowledge in the contexts of truth and power (Foucault, 1980), is to engage in “a persistent critique of what one cannot not want” (Spivak, 1974, p. 28).²

If it is that the postmodern perspective interrogates the ontological status of knowledge (Dewey, 1929; Foucault, 1980) and intends to disrupt metanarrative³ (Lyotard, 1984; St. Pierre, 2000; Walkerdine, 2004), then it must be for this traditional perspective of mathematics to be challenged. Constructivism, as a postmodern epistemology⁴ (Glaserfeld, 1995; Noddings, 1990), “breaks with convention and develops a theory of knowledge in which knowledge does not reflect an ‘objective’ ontological reality, but exclusively an ordering and organization of a world constituted by our experience” (Glaserfeld, 1987, p. 199). A postmodern perspective on knowledge undermines a traditional view of the world, in particular mathematics and mathematics education, and above all the “relation of knowledge and reality” (p. 193). For constructivists, the word ‘knowledge’ refers to a commodity that is radically different from the objective representation of an observer-independent world that the mainstream of the Western philosophical tradition has been looking for. Instead ‘knowledge’ *refers to* hypothetical conceptual structures⁵ that appear to an observer to be viable for the learner, given the learner’s range of present experience within their socio-historical context of thought and language. The constructivist’s mathematics must reconcile this notion of the temporary ways of knowing attributed to any knower, with some sociology of knowledge—that which has been named and referred to as knowledge, i.e. “Mathematics”, by some particular group with the power to do so.

² “The only truths we can know for certain are those we have invented ourselves. Mathematics is surely the greatest of such inventions” (eighteenth century philosopher Giambattista Vico).

³ “Simplifying to the extreme, I define postmodern as incredulity toward metanarratives. This incredulity is undoubtedly a product of progress in the sciences: but that progress in turn presupposes it” (Lyotard, 1984, p. xxiv).

⁴ “The constructivist assumption should be followed by a break with epistemology.... It recognizes... the temporality of knowledge, and the existence of multiple selves behaving in consonance with the rules of various subcultures” (Noddings, 1990, p. 12).

⁵ Such conceptual structures are hypothesized by an observer to be the ways of knowing or ways of thinking of another. This thing, mathematical knowledge, is a construct of the observer.

A Constructivist Theory for Knowledge

Constructivism, as a theory for knowing and learning, brought the postmodern turn to the discussion about what it means to know mathematics, how does one teach mathematics, and what are the goals of teaching mathematics; possibly doing so by asking the question anew, what is mathematics? A constructivist perspective on knowing and learning places the learner as an agent in the world. This not only reflects an ontological turn—reflective of postmodern thought, but also an axiological shift from the behaviorist perspective that the learner is shaped by the world. Constructivism's primary turn from the established behaviorist orientation to learning was to embrace the idea that the thinking mind could be considered, or at least modeled, while behaviorism restricted itself to considering only observable behavior (Glaserfeld, 1995, 2007). The behaviorist orientation left the mind as a black box, examining inputs and outputs, while the constructivist set out to create models for what might be going on inside that black box. Constructivism embraced the learner as an active agent upon the world, rather than a passive recipient of the world. More specifically, the constructivist learner was imagined to either assimilate or accommodate the *attended-to* perceptions of the experienced world.

Reorienting epistemological assumptions to take knowledge as actively built up by a cognizing subject, rather than passively received, declares the knower as a self-organizing system. This position asserts the knower's cognitive structures are rational and internally consistent.⁶ The function of cognition is therefore adaptive—it serves the organization of the person's experiential world, not the discovery of some ontological reality. The *truth* of knowing is assumed by its viability in the knower's "domain of experience" (Glaserfeld, 1995, p. 14).

⁶ This standpoint then foregrounds that in observation, one may perceive inconsistency or contradiction in another person's cognitive structures.

Taken as such, this assumption encourages valuing each person's experiential world as serving that person.⁷

The concept of *objectivity* also takes on new meaning given the constructivist epistemology. In the interaction between two people, each person builds a model—a way of knowing—for the topic of the interaction, as well as a model for the ways of knowing and thinking of the other person in the interaction. When input from that second person seems to confirm the knower's model of the topic of the interaction, an agreement appears to be reached—a mutual understanding, a shared knowing. Feedback from the other has a confirming quality that leads the knower to have a certain confidence in her knowing. Glasersfeld calls this knowing *intersubjectivity* and considers it to be the “highest, most reliable level of experiential reality” (Glasersfeld, 1995, p. 119). This intersubjective knowing is the constructivist's objectivity.

Much of mathematics education embraced the constructivist view through promotion of the child as an active learner, evident in policy documents of the early 1990's such as United States' National Council of Mathematics Teachers' *Curriculum and Evaluation Standards for School Mathematics* (1989) and those of other countries including Israel, Japan, China, Egypt, Canada, and South Africa (Malloy, 2002). Yet tensions remained about the status of knowledge the constructivist viewpoint suggested; specifically, that as a constructed way of knowing the experiential world, the truth of such constructed knowledge was in no way determinable (Glasersfeld, 1995, 2007).

The emergence of new epistemologies coincided with the countercultural swing of the late 1960's in Western cultures. At this time constructivist ideas for education (Glasersfeld, 1975; Papert, 1980; Piaget, 1970; Vygotsky, 1978; Wittrock, 1974) took seed, liberatory and democratic movements in education found voice (Freire, 2002/1970; Illich, 1971; Kozol, 1972),

⁷ This stance is not intended to suggest passivity in interaction: as observers, and particularly as teacher, we construct second-order models of others and attempt to provoke others into disequilibrium.

and postmodern deconstructions of truth, power, and knowledge (Feyerabend, 1975; Foucault, 1982/1972) emerged. That similar goals for education grew out of each perspective, is unsurprising. Viewing children as authors of knowledge, and to imbue the child with such authority embraced the postmodern notions of power relations. Yet, save for the early levels of schooling, institutionalized education seems lost on how to proceed in a post-knowledge world. The unwillingness to relieve mathematics education from the encumbrance of an ontological existence to mathematics, has allowed for the unjust stratification of students that is at present the great challenge to cries for equitable educational outcomes, such as “Mathematics for All” (Martin, 2003). The privileged knowing ascribed to certain people would not be possible if all learners were conceived as constructors of mathematics and/or mathematical ways of knowing the world.

The Need for the Other

Although constructivist epistemology has been painted as overly individualistic (Philips, 1995; Lerman, 1996, 2000), it can form the basis for a model of ethics (Glaserfeld, 2000), and in particular a model for an ethics of liberation (Hackenberg & Lawler, 2002). By virtue of interactions, groups of autonomous beings attain intersubjective agreements that include codes or rules to guide behavior. As such, Thompson (2000) argues that constructivism predicts the emergence of ethics. Following Kant’s (1788/1938) categorical imperative, I base this discussion of ethics in establishing a need for the Other.

Social Interaction

The perceived confirmation or disconfirmation of one’s knowledge in social interaction plays a crucial role in a person’s construction of her experiential reality. By interacting with others, people have opportunities to increase the viability of their ways of knowing. Such opportunities cannot occur without interaction since a knower remains internally consistent within their own frames of reference. In other words, there is no reason, without interaction, for a person to change. “We all depend on relationships for our survival” (McKeith, 2011, p. XXX

[paragraph 3 of *Visibility and Connection*]). Since by virtue of living, a person continually interacts with an experiential reality consisting of both human and non-human entities, a person's ways of knowing are always in a state of flux and that person's viability has the potential to increase. This sort of social interaction allows for the on-going modification and stabilization of a person's ways of knowing the world and others in it. An awareness of this interplay among knowers allows one to feel "...justified in speaking of 'confirmed facts', of 'society', 'social interaction', and 'common knowledge'" (von Glasersfeld, 1995, p. 120). This reconceptualization of objectivity as intersubjectivity calls for a consideration of others in the construction of each person's experiential reality. In fact, not only does it consider others, an individual is reliant on the feedback (nay, kick back) of the Other—the unexpected response that autonomous knowers provide. Such a response is unexpected in the sense that current ways of knowing would not have predicted it.

Interaction and Ethics

As does knowledge, ethics grows from relationships between people (Neyland, 2004). In fact, without beings other than ourselves in the world, beings that we regard as autonomous and rational, ethics would not necessarily arise. "Ethics arises from an encounter with the Other, who is totally, and infinitely, Other than the self" (Atweh, Bland, & Ala'i, 2011, p. XXX [paragraph 3 of *An Approach to Ethics*]). Lewin (2000) speaks of this notion in terms of continuity. That is, if humans did not construct continuous time and space and thereby have the ability to recollect past experience, there would be no need for ethics. Since humans do recollect past experience, relationships between people and objects can be established and do change over time.

A common definition for ethics is a code of principles and rules that guide the actions and behavior of members of a group. Thus ethics involves a choice of goals for how to act, and such choices rarely are clearly defined as right or wrong. When members of a group agree, explicitly or implicitly, about these choices to guide behavior, it could be said that they have reached an intersubjective agreement; structures for how to act within their community are

taken-as-shared.

A fundamental problem in ethical philosophy is the “justification of the basic precept that the subject must adapt his or her own interests to the interests of Other” (Glaserfeld, 1985, p. 99). For example, what is the rationale for following Kant’s (1938/1788) categorical imperative to act only in ways that we wish for others to act? Kant proposes that we must consider others in our actions, but it remains questionable why we need to do so.

A person’s ways of operating won’t necessarily become more viable except through interaction—since people are internally consistent, there is no reason to change without interaction. In particular, through the development of intersubjectivity, a person’s ways of operating can become more viable because developing intersubjective agreements with others open the way to considering alternative viewpoints. Logically then, greater viability cannot be attained without conferring an independent existence to others and working to conceptualize others’ ways of knowing. Hence, the constructivist theory for learning and knowing posits an inherent need to consider others, a need that resides below consciousness. This need for the other provides fertile ground from which a model of ethics should emerge. Such a postmodern ethics helps to define a mathematics education for social justice.

A Framework for Mathematics Education for Social Justice

Although there have been multiple definitions for what it means to teach mathematics for social justice (Burton, 2003; Gutstein, 2006; Gutierrez, 2007; Keitel, 1998; Povey, 2002; Sriraman, 2007), here I suggest four cornerstones that characterize the teaching of mathematics for social justice, in particular that each of the four must be considered: *access*, *achievement*, *authority*, and *action*. For me, social justice in mathematics education does not end with attaining greater access to or achievement in mathematics, education, or the larger culture. The notions of *authority* for knowing and both the confidence and compulsion to *act* are of equal or possibly greater status when devising a notion of mathematics education for social justice.

I will not attend to the necessary cornerstones of equitable access⁸ and achievement⁹ in this discussion of developing a social justice perspective; the lack of attainment of these goals has been identified, decried, and deconstructed to a point of exhaustion. The field of mathematics education has a long history documenting such iniquities (Lawler, 2005), and there seems to be an adequate sense of what *could be* or *ought to be* done, or at least a great many opinions and ideas—a “discourse of moral judgments” (Atweh, Bland, and Ala’i, 2011, p. XXX [last paragraph before *An Ethical Basis...*]). But rather than focusing on these iniquities, Gutierrez’ gap-gazing (2008), I hope to elaborate on notions of authority and action, especially in the context of a constructivist epistemology.

To set a stage, I draw upon constructivist tradition to recognize a *children’s mathematics* (Steffe, 2004), that which I as a teacher assume a student to have constructed; the mathematical ways of knowing and ways of thinking that I attribute to the child. For the sake of the remainder of this paper, I will refer to such mathematics as lower case (m)athematics when the specificity of meaning warrants distinctions.

Continuing, *mathematics for children* are an adult’s ways of mathematical knowing and thinking (Steffe, 2004), drawn upon in order to hypothesize a zone of potential construction (Steffe & Thompson, 2000b) for directing interaction with a child. Although still always a constructed knowledge, we as teachers treat this sort of mathematics, that which appears in textbooks and curriculum guides and standards documents, “the race-expression embodied in that thing we call curriculum (Dewey, 1902, p. 31), as what is to be learned in the classroom. This particular mathematics, a mathematics for children, will be referred to with an upper case (M)athematics. It is an idea similar to what is often called School Math; however my

⁸ Access as *opportunity to learn*, even if identified long ago, remains a significant gatekeeper (Moses & Cobb, 2001) whether blatantly or in more nuanced forms (Nasir & Cobb, 2007).

⁹ Gutierrez (2002) calls for a fine-tuning of the concepts *access* and *achievement* as markers of a just mathematical education. Acknowledging heterogeneity within and between groups of students, it is not evident that having all students reach the same goals represents justice for students’ own desires or sense of self. Rather, she emphasizes a goal of being unable to predict patterns in achievement “based solely on characteristics such as race, class, ethnicity, sex, beliefs and creeds, and proficiency in the dominant language” (p. 153).

characterization reflects the constructivist's ontological status of knowledge, positioning (M)athematics in the adult's ways of knowing.

The (M) / (m) distinction allows for further discussion of access, achievement, authority, and action. The notions of access and achievement are fully in relation to (M)athematics. Ernest (2002) names this *mathematical empowerment*. This privileged power/knowledge, an enlightenment era relic, remains a gateway (Moses & Cobb, 2001) to the cultural capital that schools are directed to deliver. Gutstein (2006) noted that a teaching goal for mathematics must embrace this potential to read the (M)athematical *word*, quite similar to his teaching goal to succeed academically in the traditional sense. Gutstein extended this argument that mathematics education should embrace the goal to read and write the *world* with mathematics;¹⁰ however he did not note the constructivist distinction among ways of referring to mathematics as I have brought forth here.

To recognize that the child both *writes the word* of (m)athematics and *writes the world* with (m)athematics (Freire, 1970/2002) is fully imbuing the learner as an author of their experiential reality, the third cornerstone for teaching mathematics for social justice. The child is an author of (m)athematics, and an actor upon the world with her (m)athematics. To both attribute this authority to the child, as well as foster the child's own awareness of this authority is the deference of power¹¹ the constructivist epistemology allows for. The child that sees oneself as the constructor of the knowledge guiding her way of knowing the world, gains an ownership in the activity of living/interacting. This sort of self-concept in relation to mathematics may be considered as a "robust mathematics identity" (Martin & McGee, 2009, p. 233). It reflects Ernest's (2002) *epistemological empowerment*, an "individual's growth of confidence not only in

¹⁰ Ernest's (2002) *social empowerment*.

¹¹ "Power is always present" (Foucault, 1997/1984), and as such, "there is freedom everywhere" (St. Pierre, 2000, p. 490).

using mathematics, but also a personal sense of power over the creation and validation of knowledge” (p. 9).

Assuring confidence and competence in the learner’s own ways of knowing and thinking are essential, but incomplete without the coupling of an awareness of the perpetual incompleteness of these ways of knowing.¹² Taken to an extreme may render the mathematical learner overly egoist, unphased by the knowing of others (Grieb & Easley, 1985; Lawler, 2008, 2010). Coming to value others’ confidence and competence in their knowing, and regarding that knowing of the other as not identical to one’s own is necessary (Lawler, 2005), what Boaler (2008) refers to as a *relational equity*. I consider this to be conferring an independent existence on others. Regarding others’ ways of knowing and thinking as not identical to one’s own always keeps in play possibility.

This shift in authority of knowledge, from the presumed guild of (M)athematicians, the (M)athematics teacher, or (M)athematics textbook, to the constructing knower, justifies the subconscious need to act upon the others that constitute one’s society in a more just manner. This need to act, to write the world, is the call for social action that underlies Gutstein’s (2006) theory. That one *does* author knowledge, mathematical or otherwise, places the knower at the foreground of the world that unfurls in front of them. We know the world, the experiential world of constructivism, through our interactions with it. Inasmuch, we have a role in shaping that world. Through our (m)athematics, we act upon the world. To engage students in reflection, discussion, and decision on intentional acts and non-acts upon the world engages them in the

¹² The notion that each person develops her own thinking does not imply separation from communities or deny the value of having some sort of shared mathematical knowledge (Hackenberg & Lawler, 2002). Knowing others are essential for developing each person’s own ways of thinking and understanding. For example, fostering the development of students’ unique directions in mathematics does not imply each student should or even could work in an isolated way. In fact, collaborating to develop knowledge that people act upon as if it were shared is essential (Steffe & Thompson, 2000a) for each person working to solve a mathematical problem. That is to say, when groups of students work on a problem, each will be developing her own way of thinking about the problem, drawing upon her own ways of knowing. As a student voices her ideas, responses from others (whether confirmatory or contradictory) will provoke her to further shape her ways of thinking. And as such, each person’s direction is connected to the ideas of others via intersubjectivity. In this way, each person’s unique direction is dynamic and relies on constructing an existence of others independent from the self.

ethics of determining and enacting what is fair, a fundamental activity of social justice. That children understand their role in authoring (writing) the world, and their decisions on how that authoring shapes the world, speaks to the fourth component of social justice education, *action*.

Teaching Mathematics for Social Justice

Toward creating a pragmatic framework for teaching mathematics for social justice, I simplify the work of teachers considering three pillars: defining curriculum, determining ways of acting in the classroom—pedagogy, and planning for activities of assessment.

Curriculum

As a postmodern orientation to knowledge, a constructivist viewpoint helps distinguish two sorts of curricular goals in mathematics education. The first can be thought of in some ways as historical study,¹³ that there is a particular (M)athematics to be learned. Secondly, the constructive activity of the learner, that activity that we, as teacher-observers may deem mathematical, must also be developed. This is (m)athematics, the ways of knowing and thinking that the teacher could not know, but attributes to the child. Here, one could say there is both a need to educate¹⁴ the child through teaching the discipline.

To ensure clarity regarding this distinction intended with the naming of (M)athematics and (m)athematics, I recall the mathematical *ways of understanding* and the *ways of thinking* (Harel, 2008) presented earlier. These qualities serve to characterize some sort of difference between knowledge that has a fact-like quality, and a knowledge that shows itself in practice. Yet the notion remains based in a worldview in which knowledge as an a priori existence; it reflects some ontological truth. I go further when stating there is a (M)athematics to be learned than simply capturing it as some univocal, hegemonic School Mathematics, in this (M)athematics attains the post-epistemological view toward knowledge. This (M)athematics, as

¹³ In consideration of the justification question (Stanic, 1984), “Why teach mathematics?” in a twenty-year retrospective (Lawler, 2005), George Stanic concludes mathematics seems to be an “interesting phenomenon that has arisen among human beings, and thus worthy of study because it’s such an important part of life, *historically*” (p. 35, italics added).

¹⁴ Note the root of educate is *educere*, meaning to draw out.

well as (m)athematics, each would have associated ways of understanding and ways of thinking.

With some greater specificity as to what might constitute the context or problem base of such a curriculum, it would be ignorant not to consider the tremendous problem sets assembled across centuries. Some of which follow an intentional trajectory toward a particular mathematics; others of which are intentionally organized around the (M)athematics a teacher posits a child may be prepared to engage. It is these that offer a more equitable potential, in that the design element is meant to start with the child's ways of knowing, (m)athematics. It may also be useful to consider the potential of a culturally-based (Ladsen-Billings, 1994; Lipka, Yanez, Andrew-Ihrke, & Adam, 2009) set of materials. Although mathematics ought to often be context for itself, often times an image or better a manipulatable object provides the mind something to operate on. This image or object is likely to be more powerfully known if it has a cultural relevancy for the child. More powerfully, a teacher may take on the challenge of developing curriculum in conjunction with the student, both overtly by inquiring to interests and pursuing seeming injustices in the students' spaces of living (Gutstein & Peterson, 2005), and covertly by predicting what mathematical prompt might create a disequilibrium in children's ways of knowing (Duckworth, 1996; Steffe, 2007).

Pedagogy

Clearly the curricular principles set forth above, being of focus on mathematical ways of knowing and thinking, overlap with pedagogical priorities. A pedagogy intended to ensure that students "have an ownership of their knowledge—a sense that it is empowering and liberating" (Ladsen-Billings, 1994, p. 77) "involves students in the knowledge-construction process" (p. 77). Culturally responsive teaching is liberating (Lipman, 1995); it guides students in understanding that no single way of knowing, no single *truth*, is total and permanent. It does not solely prescribe to mainstream ways of knowing (M)athematics, but recognizes many mathematics, and embraces the child's own (m)athematics. Such an empowering pedagogy could be

considered while engaging learners in the fabrication of both (m)athematics and (M)athematics. This emphasis clearly positions an authority for knowing squarely in the learner.

It is tempting, however, to take the name Ladsen-Billings (1994) suggests for such a pedagogy, *culturally relevant* teaching, or Gay's (2000) *culturally responsive* teaching and extract an algorithm for teaching children of some specific group (cf. NCTM's series *Changing the Faces of Mathematics: Perspectives on... [Gender, African Americans, ...]*). This sort of simplification perpetuates the marginalization and essentialization of members of these groups (Lawler, 2005). Gutierrez (in press) identifies this danger in trying to define pedagogies that take inherently raced, gendered, or classed positions by demonstrating that they reify an unstated hierarchy. But rather than attempt to extract a pedagogical algorithm, a culturally responsive pedagogy uses the cultural knowledge, prior experiences, and performance styles of diverse students to make learning more appropriate and effective for them. That is to say, this pedagogy uses the learners' life and living experiences to teach mathematics.

Gutstein's (2003) work with low-income Mexican and Mexican American students and families demonstrates the same goal, more explicitly naming a social justice pedagogy in mathematics education. "An important principle of a social justice pedagogy is that students themselves are ultimately part of the solution to injustice, both as youth and as they grow into adulthood. To play this role, they need to understand more deeply the conditions of their lives and the sociopolitical dynamics of their world" (p. 39). This basis for teaching students to study their community, to develop a sociopolitical consciousness, engages children in using mathematics to understand their lives; they are *reading the world* with mathematics. Further, his students learned to use (M)athematics to act upon their world, to analyze, critique, and advocate for themselves. In this way, a pedagogy for social justice is empowering (Shor, 1992), "a critical-democratic pedagogy for self and social change.... [It] does not teach students to seek self-centered gain while ignoring public welfare" (pp. 15-16).

A social justice pedagogy in mathematics education ought also to promote caring and compassion among students (Boaler, 2008; McKeith, *this book*). Coming to develop a mathematical empathy, an appreciation of the diversity among ways of thinking, extends authority for knowing in an ethical manner, to recognize that this Other thinks/knows in ways different from one's own. Boaler (2008) found such a pedagogy in place in an urban classroom in the US, in which a particular model for instruction was applied. In this model, Complex Instruction (Cohen & Lotan, 1997), the teacher attends to students' positioning of mathematical authority and works to disperse authority among students (Cohen, 1994). The attempt to disperse authority is one defining pedagogical move of the teacher for social justice. Complex Instruction, as a pedagogy, is designed to counter social and academic status differences among students, built upon the sociological theory that status differences emerge in group interactions, not because of particular students. It relies on four pedagogical intentionalities, developing a *multidimensional classroom*, the application of *multiple ability treatments*, *assigning competence*, and the use of *roles*—emphasizing that all students have important work to do in order to function successfully with others.

Assessment

Assessment is really not a pillar on its own, but truly a routine aspect of sound pedagogical practice (Hodgen & William, 2006). Rather than assess in order to determine what the child cannot do—a orientation toward deficiency (Lee, 2003), assessment must have as its purpose the goal to build models of what a child knows and can do, the *(m)athematics of children*. *Ranking* another's way of knowing—that is, dismissing another as “wrong” or “naïve,” or alternately considering her to be “correct” or “perfect”—involves a failure to attempt to understand the knower's experiential world and the assumptions of that person. Such behaviors are not congruent with a teaching embodiment of constructivist epistemological principles. This manner of assessment serves to “pathologize those pupils who do not succeed in [these] examinations” (Cotton, 2004, p. 227).

However, *judging the degree of viability* of another's way of knowing frees an observer (teacher) from a stance of evaluating for correctness while freeing the observed (student) from being classified. By entering the student's world and taking on her assumptions as fully as possible—obtaining an epistemological empathy—the teacher can point to ways the student operates that she may not be aware of, can question an assumption or conjecture, or make explicit a pattern of thinking. In this manner, the teacher is assessing so as to determine how to act.

In doing so, the teacher has moved from acting within classroom interactions to becoming an observer of them, what might be considered the distinction between pedagogy and assessment. This role as observer of children's (m)athematics can expand further to include consideration of what the teacher understands about what the student *could* understand (Thompson, 2000, p. 303). Intentionally constructing *and* valuing the other's way of knowing as an outgrowth of that other person's experiential world provides the basis for ethical interaction; it promotes an internal locus of authority for the learner.

While teaching to develop both (m) and (M) mathematics, the teacher's assessment practices must follow children's activity with an effort to understand the ways in which the child thinks and understand. Such assessment practice allows the teacher to make productive decisions "*to determine the environment of the child, and thus by indirection, to direct*" (Dewey, 1902, p. 31) children toward a (M)athematics intended to be taught.

The teacher assesses in order to direct, even if by indirection, the child. This conscious pedagogical effort, to assess and direct, speaks to the emergence of the curriculum. Although the (M)athematics curriculum may be named prior to the teacher-student interaction, as a set of standards or by the sequence of activities prescribed by a textbook, the actual curriculum emerges from these classroom interactions. If these mathematical interactions, between teacher and student, makes possible for the child to assert his present powers, exercise his present capacities, and realize his present attitudes (Dewey, 1902), the mathematical development of

the child could not be prescribed; it—(m)athematics—could not be known before it “appears” in interaction. And then only emerges as *mathematics of the child*, an observers model for the mathematical ways of operating attributed to the learner (Steffe, 2004). Dewey’s concluding observation, “The case is of Child” (p. 31) is then to say; there is no getting around or free from the child’s ways of thinking and ways of understanding. It is she who makes the mathematics she learns.

While (M)athematics may guide the teacher’s curricular decisions, socially just educational principles recognize that the constructing mind fabricates an internally consistent (m)athematics. I take this constructivist orientation to be my underlying premise for a socially just mathematics education. The math educator teaching for social justice further ensures that a critical recognition of a (M)athematics also develops along with a drive to critique the world with one’s own (m)athematics. In sum, access and achievement in (M)athematics, a personal authority for the creation and validation of mathematics, and that with mathematics they interact with and act upon the world, intentionally.

Knowledge as Fabrication

In sum, this postmodern, post-epistemological, post-knowledge¹⁵ framework for a mathematics education draws upon the demand for attention to access, achievement, authority, *and* action. The constructivist perspective redefines what access and achievement might be by wondering *relative to whose mathematics?* It repositions authority and authorship to take center stage. And it closely binds the responsibility for social action as inherent in each of these first three cornerstones. Acting upon ones world is how we come to know it. Further, one needs

¹⁵ We live now in a post-knowledge society. Previously, knowledge had value experts created knowledge and innovation, and the management of knowledge (i.e. intellectual property) was important. The Knowledge Era was defined by knowledge possessing some economic worth. Now, because knowledge is readily-accessible, for example through mobile devices ability to access the world wide web, the value of knowledge is near zero. In a post-knowledge society, it may be that wisdom has a new value. Wisdom comes through experience; wisdom emerges in collective knowledge, in community, in empathy for other’s ways of knowing.

healthy cohabitants in this experiential world to feed back into one's knowing of this world, populated by knower's other than oneself.

Mathematical learners, as do all learners, fabricate knowledge, where fabrication is taken to mean build, design, construct. Although the field of mathematics education seemingly has embraced the constructivist notions of the active learner and the constructing mind, it is most certain that a "softer" (Larochelle & Bednarz, 2000, p. 3) constructivism is enacted in schools, where the modernist truth agenda remains cemented in place. While the child's active mind may be increasingly valued in policy documents and students' points of view elicited in the classroom, such elicitation only serves to determine what is "wrong" about the students' way of thinking or understanding. Wrong, used in this manner, is meant as taken from the perspective that there is a pre-existing knowledge—mathematics, a truth-regime—that is to be taught. In the soft version of constructivism, the fabrication of knowledge takes on a different meaning; the knowledge fabricated by the learner is a concoction, an invention, a forgery. In essence, the soft constructivism encourages a perspective toward the learner as to be one who constructs untruths, one who fabricates lies. Without question, this is an unjust and unethical perspective to take toward another, an autonomous constructive knower other than oneself.

The aforementioned political and social ramifications for a constructivist view on learning, and the related constructed view of knowledge, have yet to be enacted in mathematics classrooms, nor taken seriously when conceiving of the activity of or goals for mathematics education. The current treatment of children as fabricators of knowledge, as little liars, may in fact be a greater injustice to the learner than teaching with the intent to deposit knowledge into the knower's mind, paraphrasing Freire's (2002/1970) banking model. In this insidious current model for teaching young mathematical fabricators, we engage them in activity, engendering them with a momentary belief that we are truly interested in what they are thinking about their world. And then we tell them how it truly is, how they should have figured, how they should think. We not only continue to act in accordance with a belief that language may somehow

transmit knowledge, an illusory notion (Glaserfeld, 1998), but we enforce and enhance the modernist knowledge-as-truth agenda onto the mathematical learner.

When unquestioningly engaged in an epistemology of soft constructivism, we treat the learning activity as a process of discovery, holding tight to a knowledge that is to be discovered, listening *for* (Davis, 1997) cues to hear in the child our own ways of knowing this knowledge, (M)athematics. The pedagogical practices of the teacher devolve to a guess-what-I'm-thinking state; the pressure of time and the testing of this pre-existing knowledge drive the maddening process of an education that began with a hopeful premise—that children make meaning through active engagement with their experiential world, that children are knowledge constructors, fabricators.

If the radical epistemology of constructivism is embraced and the fabrication of mathematical knowledge is recognized not as a construction of untruths but as other truths, as each learner's ways of thinking and understanding, a different mathematics education must be conceived. Such a mathematics education would mature from this postmodern constructivist epistemology, and its concordant poststructural concept of power/knowledge (Foucault, 1982). Such a mathematics education would be ripe to more powerfully embrace the socially just calls for access, achievement, authority, and action. Furthermore, this ethic for interaction, in particular an intentional teaching interaction, is not only more just, but also brings forth a relational equity among students.

Conclusion

In the postmodern, knowledge may remain a sort of commodity. If so, Foucault's power-knowledge positions the two in relation much more closely than the simple notion that "knowledge is power" so as to recognize the bidirectionality of both the disciplinary control of knowledge and the assertion of knowledge to control. So what role mathematical knowledge in the teaching and learning of mathematics for social justice? I bring forth Dewey once again to close with some sort of answer to the question. (M)athematical knowledge is that race-

expression (Dewey, 1902) of man. The ways in which the teacher thinks and knows of this (M)athematics defines a sort of map. “[I]t gives direction [and] it economized effort,... pointing out paths which lead most quickly and most certainly to a desired result” (p. 20). As a map, (M)athematical knowledge serves the teacher in making decisions about “what there is in the child’s present that is usable with reference to it [(M)ath]” (p. 23). This allows the teacher to know what problem to pose, topic to explore, puzzle to convey. It is the pedagogical practice of the teacher to make these curricular decisions, based on assessment of the student’s present ways of (m)athematical knowing. The teacher sees to it that “day by day the conditions are such that *their own activities* move inevitably in this [(M)ath] direction (p. 31). The pedagogical decree, that the teacher is to serve as a recurring guidepost on the (M)athematical map, speaks especially to the role of authority in a mathematics education for social justice. The map is not the territory of the student’s (m)athematics. To spark and value this (m)athematics is *just*; to engage a classroom to come to know others’ (m)athematics creates *relational equity*; and to understand that one implicitly acts on the world with (m)athematics and feels empowered to do so with (M)athematics enhances personal authority, ethical relationships, and a commitment to act.

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